

MODEL OF TWO-PHASE FLOW WITH COAGULATION  
OF PARTICLES OF A POLYDISPERSE CONDENSATE  
IN THE ONE-DIMENSIONAL APPROXIMATION

A. N. Kraiko and A. A. Shraiber

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We shall discuss various questions connected with the construction of models, describing the flow of a mixture of a gas and a polydisperse condensate with allowance for coagulation of particles. The one-dimensional approximation is used with the effects of viscosity and thermal conductivity of the gas being taken into account only in the description of the gas-particle interaction. The present approximation corresponds, in particular, to the case of flow in Laval nozzles, where the difference of the Reynolds numbers, determined by a characteristic nozzle dimension (e.g., the radius of the minimum cross section) and by the particle diameter, as a rule, justifies such an approach outside a comparatively thin boundary layer at the nozzle wall. Special attention is given to the redistribution of the momentum and energy of the particles, produced during coagulation. In addition, a number of points of a methodological nature, in the opinion of the authors not finding sufficiently complete elucidation in the available literature on coagulation [1-9], are refined.

In the literature on coagulation published at the present time, the redistribution of the momentum and energy of the particles is either not considered at all (cf. [1, 3, 6], in which equations are obtained, determining only the evolution of the fractional composition of the condensate), or it is postulated that the momentum [2, 5] or momentum and energy [4, 7-9] of a particle, formed during coagulation, are uniformly distributed among all particles of a given size. Such an assumption either should be considered as the result of an averaging procedure (over the velocities and energies of particles of one size), which is not in itself rigorous, or, as noted in [10], supposes the existence of some sort of mechanism for exchange of momentum and energy between particles of one size, which could only happen with sufficiently frequent collisions of the indicated particles, not, however, leading to coalescence, and this can hardly be considered practicable. Therefore, a complete solution presupposes the introduction of velocity and temperature distribution functions for each mass interval along with the distribution function over particle size, as is done in the absence of coagulation, e.g., by Williams [11]. Without writing out the equations so obtained, we note that the resulting system becomes commensurate in complexity with (or even exceeds) the system of equations in the kinetic theory of gases. This circumstance makes the practical use of coagulation models with multidimensional distribution functions somewhat problematic in the near future at least.

In connection with what was said above, it seems reasonable to consider along with the well-known models (e.g., the one adopted by Grishin et al. [4]) a model built on the assumption that the excess (or deficient) momentum and energy, originating with the particles produced as a result of coagulation (or some part thereof), is not distributed uniformly among the particles of a given mass interval but rather is transferred to the gas. The validity of the assumption that the excess momentum and energy is completely transferred to the gas for particles of sufficiently small size is corroborated by the following considerations.

Let the dynamic and thermal relaxation times of the particles  $\tau_f$  and  $\tau_q$ , which in the Stokes approximation are proportional to the square of the particle diameter, be small in comparison with a characteristic flow time (the latter in the stationary case is defined as the ratio of a characteristic linear dimension to a characteristic velocity). Then, as shown, e.g., by Rannie [12], in the absence of collisions between the particles the particle parameters are almost everywhere very close to the corresponding gas parameters, differing from them by magnitudes on the order of  $\tau_f$  and  $\tau_q$ . Moreover, the coefficients of the correspond-

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ing expansions in  $\tau_f$  and  $\tau_q$  are proportional to the first derivatives of the gas velocity and temperature with respect to the spatial coordinate. Thus, with an accuracy to  $\tau_f$  and  $\tau_q$  inclusive, the parameters of particles of one size almost everywhere do not depend on the previous history of the motion of an individual particle and are determined by the gas parameters and their first derivatives at the point of interest. If, however, for some reason or another (e.g., due to external action, collision with other particles, etc.) the parameters of a particle should be appreciably different from their "quasiequilibrium" values, then by definition of  $\tau_f$  and  $\tau_q$  the process of smoothing out the velocity and temperature of such a particle takes place over lengths on the order of  $l_f = w_s \tau_f$  and  $l_q = w_s \tau_q$ , respectively (here  $w_s$  is the particle velocity). We note, incidentally, that in many cases of interest in practice  $\tau_f$  and  $\tau_q$ , and consequently  $l_f$  and  $l_q$ , are close in order of magnitude.

In addition to the above let  $\tau_f$  and  $\tau_q$  be small also in comparison with the mean free time of a particle of a given mass interval. Then the velocity and temperature of a particle, produced by the merging of smaller particles, will differ from the corresponding parameters of the particles of the same mass interval, produced significantly earlier, only over the length  $l_f$  and  $l_q$ , which are small in comparison with a characteristic flow length and with the mean free path of the particles  $l$ . Over the indicated lengths the excess (deficient) velocity and energy of the particles are transferred (supplied) on account of the interaction with the gas, which leads to a smoothing of the parameters of the particles within each mass interval. Herewith the gas experiences an additional force and energy effect from the particles. We note that in the absence of coagulation the above mechanism was considered by Marble [13] in problems of flow of a polydisperse condensate with collisions of the particles. It is to be expected that with increasing particle sizes, i.e., with increasing  $l_f$  and  $l_q$  in comparison with the mean free path  $l$  and with a characteristic dimension of the problem, the difference between the above and the actual mechanisms for redistributing the energy and momentum of the particles grows. Nevertheless, the use of this mechanism is justified even in this case, especially if we consider that the hypothesis applied at the present time that the energy and momentum is redistributed among the particles of each mass interval, can hardly be rigorously founded even in some kind of limiting cases, bearing on flow in Laval nozzles with coagulation. In such cases, however, it is expedient to use at first several simplified models (including the mode), based on the redistribution hypothesis), since comparison of corresponding results can give some idea about the order of magnitude of the errors involved in not introducing velocity and temperature distribution functions for the particles of each fraction.

1. Let us consider the flow of a mixture of gas and a polydisperse condensate in a Laval nozzle, the cross sectional area of which we denote by  $F$ . The  $x$  axis is directly along the axis of the nozzle in the direction of the flow. The fractional composition of the condensate, i.e., the distribution of the particles with respect to size (the radius  $r$ ) or mass  $m$  is determined by the number distribution function  $n(m)$ . This is introduced such that the number of particles in a unit volume with mass in the interval from  $m$  to  $m + dm$  is equal to  $n(m) dm$ . Along with  $n(m)$  we shall use the loss distribution function  $g(m)$ , which is related to  $n(m)$  by the equation

$$g(m) = mn(m)w(m) / W\rho w \quad (1.1)$$

and which when multiplied by  $dm$  gives the mass loss of the same fraction, passing through a unit cross-sectional area of the nozzle (relative to the total condensate loss  $W\rho w$ ). In (1.1)  $\rho$  and  $w$  are the density and velocity of the gas,  $W$  is a given constant, equal to the ratio of the mass losses of the condensed and gas phases, and  $w(m)$  is the velocity of the particles of the mass interval in question. An argument in parentheses will henceforth denote that the corresponding parameter characterizes particles of the given mass interval. Thus, the relaxation lengths and mean free path length mentioned earlier will be denoted by  $l_f(m)$ ,  $l_q(m)$ , and  $l(m)$ , the radius of the particles by  $r(m)$ , and the temperature, specific internal energy, and total energy of the particles by  $T(m)$ ,  $e(m)$ , and  $E(m) = e(m) + w^2(m)/2$ .

The equations describing the change as a function of  $x$  in the parameters of the particles of each mass interval (including the distribution functions) and in the gas parameters are obtained by applying the conservation of mass, momentum, and energy to the particles or to the gas in a volume bounded by two cross sections of the nozzle (at  $x$  and  $x + \Delta x$ , where  $\Delta x$  is small compared with a characteristic dimension of the problem). The flow is here considered to be stationary, and effects associated with phase transitions and with particles being precipitated out at the walls are assumed absent. In this case the conservation of mass for the particles of the mass interval  $(m, m + dm)$  is written in the form

$$\frac{\Delta [Fmn(m)w(m)dm]}{R(m)} = R^+(m) - R^-(m), \quad \Delta\varphi = \varphi(x + \Delta x) - \varphi(x) \quad (1.2)$$

Here  $R^+(m) dm$  and  $R^-(m) dm$  are the numerical rates of production and disappearance of particles of the mass interval in question per unit volume. The right side of (1.2) in accordance with the definition of  $R^+(m)$  and  $R^-(m)$  gives the change per unit time in the mass of the particles in the mass interval  $(m, m + dm)$  in the volume  $F\Delta x$ , included between the cross sections at  $x$  and  $x + \Delta x$ . The left side gives the difference of the corresponding mass fluxes, passing (also per unit time) through the indicated cross section.

The expressions for  $R^\pm(m)$  were obtained earlier, e.g., by Grishin et al. [4]; however, it is of some expediency to consider this question once more. We begin with  $R^-(m)$ , i.e., with the rate of disappearance of particles with mass between  $m$  and  $m + dm$ . Loss of particles from this mass interval occurs due to collisions and subsequent coagulation with particles of all other mass intervals. The number of collisions of a particle from this mass interval with particles from the mass interval  $(\mu, \mu + d\mu)$  in time  $\Delta t$  is equal to  $k(m, \mu) n(\mu) d\mu \Delta t$ , where  $k(m, \mu)$  is a function called the coagulation constant. For spherical particles, ignoring the effects of the curvature of their trajectories, we assume [4]

$$k(m, \mu) = \pi [r(m) + r(\mu)]^2 |w(m) - w(\mu)| \quad (1.3)$$

In accordance with what has been said, one particle from the mass interval  $(m, m + dm)$  experiences in time  $\Delta t$  a number of collisions with particles of all other mass intervals equal to

$$\Delta t \int_0^\infty k(m, \mu) n(\mu) d\mu$$

In this expression and further on, zero and infinity in the limits of integration in actuality denote the masses of the smallest and largest particles of the condensate. Hence, for the mean free time and path length  $\tau(m)$  and  $l(m)$  we obtain the expressions

$$\tau(m) = \left[ \int_0^\infty k(m, \mu) n(\mu) d\mu \right]^{-1}, \quad l(m) = \tau(m) w(m) \quad (1.4)$$

Particles of the mass interval in question traverse the interval  $\Delta x$  in time  $\Delta t = \Delta x / w(m)$ . Since  $Fw(m) n(m) dm$  of such particles per unit time traverse the cross section at  $x$ , in the volume  $F\Delta x$

$$Fn(m) \Delta x dm \int_0^\infty k(m, \mu) n(\mu) d\mu$$

particles of the given mass interval coagulate per unit time. Comparing the obtained expression with (1.2), we find

$$R^-(m) = n(m) \int_0^\infty k(m, \mu) n(\mu) d\mu$$

If coagulation occurs with every collision of the particles, and this will be assumed henceforth, then for (1.2) to be valid it is necessary for the inequality  $\Delta x < l(m)$  to be satisfied, since otherwise not only the first but also subsequent collisions of particles, already having disappeared owing to coagulation, would be included in the right side of (1.2). Since in fact (1.2) is not used, but rather the differential equation

$$d[Fn(m)w(m)]/dx = FR(m) \quad (1.6)$$

obtained from (1.2) for  $\Delta x \rightarrow 0$ , the limitation mentioned turns out to be inessential. In reality, in passing to the limit,  $\Delta x$  as usual is taken to be "physically" infinitesimally small. In the present case in accordance with what has been said, this means that  $\Delta x$  is small in comparison with  $l(m)$  and with a characteristic dimension of the problem, but much bigger than the distance between particles.

In obtaining the equation for  $R^+(m)$  it is necessary to consider collisions leading to the formation of particles in the mass interval  $(m, m + dm)$  from particles of lower mass intervals. Let  $\mu$  be the mass of the lesser and  $\mu^\circ$  the mass of the larger (possibly equal to  $\mu$ ) of the colliding particles (in accordance with this, with an accuracy to small terms of a higher order,  $\mu < m/2$ ). In order that collisions of particles in the mass interval  $(\mu, \mu + d\mu)$  lead to the formation of particles in the mass interval  $(m, m + dm)$ , the masses of the colliding particles in the  $\mu\mu^\circ$  plane should correspond to the shaded parallelogram in Fig. 1 of area  $dmd\mu$ , where  $\mu_2^\circ = m - \mu$ ,  $\mu_1^\circ = \mu_2^\circ - d\mu$ ,  $\mu_3^\circ = \mu_1^\circ + dm$ , and  $\mu_4^\circ = \mu_2^\circ + dm$ . Finding the number of collisions corresponding to particles of mass  $\mu$  and  $\mu^\circ$  from the indicated parallelogram, then integrating over  $\mu$  from 0 to  $m/2$ , and comparing the expression so obtained with the right side of (1.2), we arrive at the equation

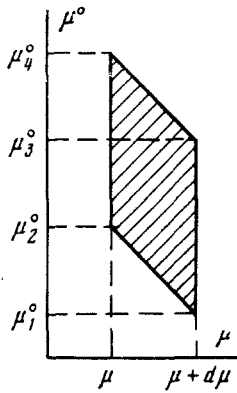


Fig. 1

$$R^+(m) = \int_0^{m/2} k(\mu, m-\mu) n(\mu) n(m-\mu) d\mu \quad (1.7)$$

We note that the present equation is the same as that given by Grishin et al. [4], and the method by which it was obtained is similar to that used by Kuentzmann [5]. In addition, in the derivation of (1.7) only pair collisions were considered, i.e., simultaneous collisions of three or more particles were considered to be unimportant. Here the number of collisions was computed by summing the number of collisions, experienced by the smaller particles [i.e., by particles in the mass interval  $(\mu, \mu + d\mu)$ ]. Of course, the same quantity  $R^+(m)$  is obtained by summing the collisions of the larger of the colliding particles. This leads to the equation

$$R^+(m) = \int_{m/2}^m k(\mu^o, m-\mu^o) n(\mu^o) n(m-\mu^o) d\mu^o$$

From here and from (1.7) it is not difficult to get the expression

$$R^+(m) = 1/2 \int_0^m k(\mu, m-\mu) n(\mu) n(m-\mu) d\mu \quad (1.8)$$

equivalent to (1.7). We note, incidentally, that by virtue of its definition the coagulation constant is symmetric, i.e.,  $k(\mu, m-\mu) = k(m-\mu, \mu)$ .

Taking into account the equation of continuity for the gas phase

$$F\rho w = \text{const} \quad (1.9)$$

and the definition of the loss distribution function  $g(m)$ , we can rewrite equation (1.6) in the form

$$dg(m)/dx = mR(m)/W\rho w \quad (R(m) = R^+(m) - R^-(m)) \quad (1.10)$$

Here the number distribution functions  $n(m)$ ,  $n(\mu)$ , ... in (1.5) and (1.7) or (1.8) can also be replaced by  $g(m)$ ,  $g(\mu)$ , ... in accordance with (1.1).

2. The complete system of flow equations along with the equation of continuity for the gas (1.9) and the equation (1.10), describing the evolution of the fractional composition of the condensate (according to the loss distribution function), includes the momentum and energy equations for the mixture components. It has already been pointed out that these are found by applying the corresponding conservation laws to the particles of each mass interval and to the gas in the volume  $F\Delta x$ . Here it is assumed, as usual, that between the gas and the particles there takes place dynamical and thermal interactions, due to the viscosity and thermal conductivity of the gas phase. Let  $f(m)$  and  $g(m)$  be the force with which the gas acts on the particles and the heat flux from the gas to the particles per unit mass of particles of the corresponding mass interval, where, as before, the quantity in parentheses is understood not as an argument, but as an indication of the particle size (mass). We ignore the direct thermal and force interactions between the flux and the nozzle wall and also between particles of different mass intervals and restrict ourselves to the case of no external forces or energy sources. In addition, we denote by  $m[w(m, \mu) - w(m)]$  and  $m[E(m, \mu) - E(m)]$  that part of the momentum and total energy of a particle, formed as a result of coagulation of smaller particles (of mass  $\mu$  and  $m-\mu$ ), which then (over distances less than  $\Delta x$ ) is uniformly distributed among all particles of the mass interval  $(m, m+d\mu)$ .

Employing the assumptions and notation introduced above, and performing certain transformations using the above equations, we can show that the conservation of momentum and energy of the particles in the mass interval  $(m, m+d\mu)$  leads to the relations

$$\begin{aligned} w(m) dw(m)/dx &= f(m) + Q_w(m)/n(m) \\ w(m) de(m)/dx &= q(m) + Q_e(m)/n(m) \\ Q_w(m) &= \int_0^{m/2} [w(m, \mu) - w(m)] k(\mu, m-\mu) n(\mu) n(m-\mu) d\mu \\ Q_e(m) &= \int_0^{m/2} [E(m, \mu) - E(m)] k(\mu, m-\mu) n(\mu) n(m-\mu) d\mu - Q_w(m) w(m) \end{aligned} \quad (2.1)$$

where the upper limits in the integrals can be replaced by  $m$ , at the same time introducing a factor of  $1/2$  in front of the integrals, as in going from (1.7) to (1.8).

The hypothesis assumed by Grishin et al. (4) that the momentum and energy of the particles, produced as a result of coagulation, is uniformly distributed among all particles of the mass interval in question is equivalent to the identities

$$w(m, \mu) = w^\circ(m, \mu), \quad E(m, \mu) = E^\circ(m, \mu) \quad (2.2)$$

where  $w^\circ(m, \mu)$  and  $E^\circ(m, \mu)$  are the velocity and total energy of a particle, formed by the merging of particles with masses  $\mu$  and  $m - \mu$ .

If as before we assume that the colliding particles move parallel to the nozzle axis, then the conservation of mass, momentum, and energy, applied to the problem of the merging of two particles, give

$$\begin{aligned} w^\circ(m, \mu) &= \mu^\circ w(\mu) + (1 - \mu^\circ) w(m - \mu) \\ E^\circ(m, \mu) &= \mu^\circ E(\mu) + (1 - \mu^\circ) E(m - \mu) \quad (\mu^\circ = \mu / m) \end{aligned} \quad (2.3)$$

As we have already noted, the assumption that the parameters of particles in the mass interval in question are completely smoothed out, which lies at the base of the identities (2.2), is not the only one possible. If these identities are not satisfied, then this means that the excess (deficient) velocity and total energy of a newly produced particle in comparison with the corresponding quantities, describing the motion of the other particles of the given function, i.e., with  $w(m)$  and  $E(m)$ , are transferred to the gas or to the particles of other mass intervals. It can be shown that in the case when direct force and thermal interaction of the particles of different mass intervals is insignificant, i.e., when the violation of identities (2.2) is caused by the "force" interaction of the newly produced particles with the gas, the equations of motion and energy of the gas in the one-dimensional approximation are written in the form

$$\begin{aligned} \rho w \frac{dw}{dx} + \frac{dp}{dx} + \int_0^\infty m [n(m) f(m) + Q_w^\circ(m)] dm &= 0 \\ w \frac{dh}{dx} - \frac{w}{\rho} \frac{dp}{dx} + \frac{1}{\rho} \int_0^\infty m \{n(m) q(m) + n(m) [w(m) - w] f(m) + Q_e^\circ(m)\} dm &= 0 \end{aligned} \quad (2.4)$$

Here  $Q_w^\circ(m)$  is obtained from  $Q_w(w)$  by substituting  $w^\circ(m, \mu)$  for  $w(m)$ , and  $Q_e^\circ(m)$  from  $Q_e(m)$  by substituting  $E^\circ(m, \mu)$  for  $E(m)$ ,  $Q_w^\circ(m)$  for  $Q_w(m)$  and  $w$  for  $w(m)$ .

The difference in the character of the interaction with the gas of the newly formed particles and the other particles of a given mass interval is due to the difference of their velocities and temperatures, owing to which the force  $f(m)$  and heat flux  $q(m)$  for particles of the first group are considerably greater than for the second.

We note that any of the differential equations (2.4) can be replaced by the equation

$$\begin{aligned} \rho w \frac{d}{dx} \left( h + \frac{w^2}{2} \right) + \int_0^\infty m \{n(m) [q(m) + w(m) f(m)] + Q_h^\circ(m)\} dm &= 0 \\ (Q_h^\circ(m) = Q_e^\circ(m) + Q_w^\circ(m) w) \end{aligned}$$

from which one obtains the Bernoulli integral in the case of a pure gas flow. In addition, using (1.10), (2.1), (2.3), (2.4), and the symmetry property of the function  $k(m, \mu)$ , we can show that for any  $w(m, \mu)$  and  $E(m, \mu)$  the following "integral" conservation laws hold:

$$\begin{aligned} \frac{d}{dx} \int_0^\infty g(m) dm = 0, \quad \rho w \frac{d}{dx} \left[ w + W \int_0^\infty g(m) w(m) dm \right] + \frac{dp}{dx} &= 0 \\ \frac{d}{dx} \left[ h + \frac{w^2}{2} + W \int_0^\infty g(m) E(m) dm \right] &= 0 \end{aligned}$$

The fact that these laws are satisfied is indicative of the consistency of the model in question, and verifying these laws while calculating can be used for control of the accuracy of the calculations.

To the equations obtained above, it is necessary to add expressions for  $\rho$ ,  $h$ ,  $e(m)$ ,  $f(m)$ , and  $q(m)$  in terms of the other parameters of the flow. We shall assume the indicated expressions to have the form

$$\begin{aligned}
\rho &= \rho(p, T), \quad h = h(p, T), \quad e(m) = e_s [T(m)] \\
f(m) &= \varphi^1 \cdot [w - w(m)], \quad q(m) = \varphi^2 \cdot [T - T(m)] \\
\varphi^i &= \varphi^i [m, p, T, T(m), w - w(m)], \quad i = 1, 2
\end{aligned} \tag{2.5}$$

Such a notation assumes, in particular, the absence of nonequilibrium physicochemical processes in the gas phase. The right sides in (2.5) are known functions of their arguments.

Together with the equations (2.2), determining  $w(m, \mu)$  and  $E(m, \mu)$  consistent with the hypothesis adopted in [4] that the energy and momentum of the newly formed particles be uniformly distributed, the conservation equations (1.9), (1.10), (2.1), and (2.4) and the constitutive relations (1.1), (1.3), (1.5), and (1.7) or (1.8), (2.3), and (2.5) form a closed system, which for a given nozzle shape and under a number of additional conditions (e.g., for a given relative particle loss  $W$ ) completely describes the change in the flux parameters as functions of  $x$ . Here  $Q_w^\circ(m) \equiv Q_e^\circ(m) \equiv 0$  and the equations of motion and energy of the gas (2.4) go over to those obtained by Grishin et al. [4].

In the other extreme case assuming that the excess momentum and energy of the newly created particles is completely transferred to the gas, instead of (2.2) we obtain the equation

$$w(m, \mu) = w(m), \quad E(m, \mu) = E(m) \tag{2.6}$$

by virtue of which the last terms in the momentum and energy equations for the condensate vanish.

3. In reality, the equations (2.6) are never strictly satisfied. However, if the conditions in the flow are such that  $l_f(m)$  and  $l_q(m)$  are much less than not only a characteristic dimension of the problem  $L$ , but also the mean free path length  $l(m)$ , then the distinction between the real situation and that described by these equations will not be very great. We note, incidentally, that if this should take place, then in deriving the flow equations we can choose a  $\Delta x$  which in addition to the conditions stipulated earlier is greater than  $l_f(m)$  and  $l_q(m)$ . In this connection the equations giving  $l(m)$ ,  $l_f(m)$ , and  $l_q(m)$  in terms of the mixture parameters, particle size, etc. are of some interest. When obtaining these equations, we restrict ourselves to the regime of small lag of the particles behind the gas, which already assumes that the inequalities  $l_f(m) \ll L$  and  $l_q(m) \ll L$ , necessary for the validity of (2.6), are satisfied. In addition, we assume that in calculating  $f(m)$  and  $q(m)$  we can use the Stokes equations, by virtue of which the energy and momentum equations of an individual particle are written in the form

$$\begin{aligned}
l_f(m) \frac{dw(m)}{dx} &= w - w(m), \quad l_q(m) \frac{dT(m)}{dx} = T - T(m) \\
l_f(m) &= \frac{2}{9} \frac{\rho_s^\circ}{\rho} \frac{w(m)}{w} \left( \frac{r}{L} \right)^2 \text{Re}, \quad l_q(m) = \frac{3}{2} \text{Pr} c_s^\circ l_f(m), \quad \text{Re} = \frac{wL}{\nu}
\end{aligned} \tag{3.1}$$

Here  $\rho_s^\circ$  is the density of the material of the particles;  $c_s^\circ$  is the specific heat of the particles divided by the specific heat of the gas  $c_p$ ;  $\nu$  is the kinematic viscosity of the gas;  $\text{Pr}$  is the Prandtl number;  $\text{Re}$  is the Reynolds number; and  $x$ ,  $l_f(m)$ , and  $l_q(m)$  are relative to a characteristic linear dimension  $L$ . In cases when  $\text{Pr}$  and  $c_s^\circ$  are close to unity (which is usually the case), the dynamical and thermal relaxation lengths are of the same order, as can be seen from the equation for  $l_q(m)$ .

If  $l_f(m) \ll 1$ , then from (3.1), expanding  $w(m)$  in a series in powers of  $l_f(m)$ , we find

$$w(m) = w - l_f(m) \frac{dw}{dx} \tag{3.2}$$

where when determining  $l_f(m)$  from (3.1), we should set  $w(m) = w$ . Putting (3.2) into (1.3) and (1.4) gives

$$\begin{aligned}
l(m) &= 6 [W | d \ln w / dx | J(m^\circ) \langle r \rangle / L \text{Re}]^{-1}, \quad m^\circ = m / \langle m \rangle, \\
\langle m \rangle &= \rho_s / N, \quad \langle r \rangle = (3 \langle m \rangle / 4 \pi \rho_s^\circ)^{1/3}
\end{aligned} \tag{3.3}$$

Here  $N$  is the total number of particles per unit volume,  $\rho_s$  is their resulting density,  $\langle m \rangle$  and  $\langle r \rangle$  are the mass and radius of a "mean" particle,  $l(m)$  is in units of  $L$ , and

$$\begin{aligned}
J(m^\circ) &= \int_0^\infty [r^\circ(m^\circ) + r^\circ(\mu^\circ)]^2 | l_f^\circ(m^\circ) - l_f^\circ(\mu^\circ) | n^\circ(\mu^\circ) d\mu^\circ \\
r^\circ(m^\circ) &= r(m) / \langle r \rangle, \quad l_f^\circ(m^\circ) = l_f(m) / l_f(\langle m \rangle), \quad n^\circ(\mu^\circ) d\mu^\circ = n(\mu) d\mu / N
\end{aligned}$$

Owing to the definition of quantities measured in degrees,  $J(m^\circ)$  is a number on the order of unity. In accordance with (3.1) and (3.3) for the ratio of the relaxation length  $l_f(m)$  to the mean free path length we obtain the equation

$$\frac{l_f(m)}{l(m)} = \frac{W}{27} \frac{\rho_g^0}{\rho} \left| \frac{d \ln w}{dx} \right| J(m^0) \frac{\langle r \rangle}{L} \left[ \frac{r(m)}{L} \right]^2 \text{Re}^2$$

from which it is seen that the cases when  $l_f(m) \ll l(m)$ , and, consequently, equations (2.6) are valid, are quite real. On the other hand, the greater the degree to which the conditions  $l_f(m) \ll 1$  and  $l_f(m) \ll l(m)$  are violated, the less justified is the use of the mentioned equations.

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